

Robust Gaussian Teleportation with Attenuations and Non-unity Gain

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Abstract

The average fidelity of the teleportation of a coherent state is calculated for general Gaussian bipartite systems shared by the partners of the protocol, Alice and Bob. It is considered that the shared Gaussian bipartite modes suffer independent attenuations before the processing of Alice and Bob. Moreover the classical communication between the partners can be controlled by a gain not necessarily unitary. Comparing with the classical fidelity threshold of measure-and-prepare methods, we establish several genuinely quantum teleportation conditions which depend on the gain and the local attenuations. Considering that the gain can be tuned to maximize the bipartite state set able to genuinely quantum teleportation, a condition for teleportation robust to local attenuations is found. This condition is demonstrated to be essentially equivalent to the condition of robust Gaussian bipartite entanglement, obtained in previous articles, showing that the attenuation robustness is an entanglement property relevant for characterization and application of bipartite systems. For the derivation of the robust teleportation conditions, the Gaussian operations onto the bipartite system are thoroughly studied, so that the transformations that maintain the fidelity invariant are found. Some scenarios for different Gaussian bipartite states are presented and discussed.

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I. INTRODUCTION

Teleportation was one of the first proposed protocols on quantum information and it is understood as an elementary piece for more complex quantum processing and communication. Moreover, it is a resource for the understanding of fundamental issues, such as the EPR (Einstein-Podolsky-Rosen) paradox and nonlocality [1]. There are many reviews available in literature, being this research field very wide and active [2–5]. In particular, implementations of the teleportation over long distances have advanced and enlarged the scientific and technological frontier [6–11]. To perform the quantum teleportation, two communication stations in different locations, usually called Alice and Bob, share a bipartite entangled system, each one retaining a part. Then Alice combines her entangled subsystem with a signal, without knowing or directly measuring it. So Alice measures the EPR observables of the combined system and transmits the obtained classical information to Bob. With the classical instructions, Bob operates unitarily his entangled subsystem to restore the original signal. Originally, the teleportation was proposed for discrete variables systems, posteriorly devised in continuous variables [12, 13]. The first theoretical and experimental setup for continuous variable teleportation, and also the most studied, was proposed by Braunstein and Kimble (BK protocol), using phase and amplitude quadrature optical modes [13–16]. In this context, the quantum teleportation requires that the bipartite entangled system should be in an optical two-mode squeezed state. However an ideal quantum teleportation, with ideal squeezed beams, is physically infeasible, because the squeezing needs to have infinity rate. Thus, since early works, the continuous variable teleportation has been studied in realistic situations, such as finite squeezing, entangled modes subject to lossy channels [17, 18], and non-unity gains of the classical communication [19–24].

Thus, even in early teleportation studies, the general connections between the capacity to perform teleportation and the entanglement of the quantum resources have been pursued and gradually clarified. Popescu has already shown that states able to teleportation are not equivalent to states that violate Bell inequalities [25]. Braunstein et al. [19] have also shown that the entanglement for fully symmetric Gaussian bipartite systems is equivalent to the quantum regime for teleportation performed by such states. However, in that same article, the authors have presented situations in which this equivalence are not applicable to non-unity gain teleportation. After that, Fiurásek [26] have derived the fidelity of a teleportation

performed by a general Gaussian bipartite system, so that we can notice that the quantum teleportation regime is very different of the necessary and sufficient entanglement condition, considering generic shared bipartite systems. On the other hand, considering local operations to symmetrize and optimize a bipartite system, the equivalence between Gaussian entanglement of the bipartite system and its ability to perform teleportation with optimal fidelity above classical threshold was established by Adesso and Illuminati [27]. Therefore, we can notice a complex and deep connection between bipartite entanglement and its ability to perform quantum teleportation. Realistic factors, such as limited squeezing, lossy quantum channels and non-unity gain of the classical communication, make this relation more diverse. In fact, given that various recent achievements [6–11] and proposals [28–31] on quantum communication involve long distances, we must consider that the shared system for teleportation can be asymmetric and suffers losses. In addition, the effective optimization by local operations can be unfeasible, because the environmental influences and the long distance between Alice and Bob make the bipartite states not fully known. So the present paper is dedicated to detail the conditions that a general Gaussian bipartite system is able to accomplish quantum teleportation, considering the afore mentioned realistic factors. Hence, considering the gain adjustment, the fidelity can be optimized so that the teleportation is maintained in a quantum regime, robust to any local partial attenuation. The condition to such robust quantum teleportation is found and it is shown that this condition is essentially equivalent to the robust bipartite entanglement condition, found in previous articles [32, 33].

To assess the quality of the teleportation, we must use a well-known quantity, called fidelity [34]. So a natural question is how much fidelity is necessary and sufficient to characterize a genuinely quantum teleportation, that is, a protocol accomplished exclusively by quantum processes. Along the years, many different conditions of quantum teleportation have been developed for different situations. In the usual continuous variable teleportation proposed by Braunstein and Kimble, a classical measure-and-prepare strategy, simulating the same task, has a maximum classical fidelity threshold (CFT) given by $F_{\text{CFT}} = 1/2$ [35, 36]. To establish the benchmark between classical and quantum regimes, originally the teleportation was restricted to transmit input coherent states and using a unity-gain classical communication. However, more recent studies generalize the benchmark for quantum teleportation to the case of squeezed state signals [37–39]. On the other hand, other studies have pursued the case of non-unity gain classical communication [21, 22, 24]. In that case,

it is possible to show that gain tuning can improve the fidelity of the sent signal [20]. Moreover, the teleportation of the single-photon qubit using hybrid continuous variable schemes [40–42] and others variations of the basic protocol [43, 44] that use non-unity gain have been analyzed theoretical and experimentally. Keeping in the task of teleporting coherent signals, one can consider a slight change of the BK teleportation protocol, in which the Alice’s input states set, $\{|\alpha\rangle\}_A$, is sent to Bob, so that he recovers an output states set, ideally represented by $\{|g\alpha\rangle\}_B$. In other words, the signal is teleported and simultaneously amplified by a gain g . Such variant is a generalization of the BK teleportation, and it is also called teleamplification.

In order to clarify the correlation properties of Gaussian bipartite systems, in Section II, it is calculated the average fidelity of the BK teleportation of an uniform set of coherent states, considering independent attenuations in quantum channels and non-unity gain of the classical communication between Alice and Bob. In Section III. Comparing the calculated fidelity to the CFT with non-unity gain, we can find a general condition for a Gaussian bipartite system to perform genuinely quantum teleportation. From symmetry considerations, we have found other interesting teleportation conditions, since they do not depend on a full system characterization, despite not being both necessary and sufficient. From one of these conditions, we show the gain can be optimized so that the state set of the bipartite system able to genuine quantum teleportation is maximized. Thus we can establish a condition to a genuinely quantum teleportation robust to attenuations. Comparisons between the teleportation conditions and early entanglement conditions are presented in Section IV, so that the robust quantum teleportation condition is verified to be essentially equivalent to the robust entanglement condition [32, 33, 45]. Hence we establish another connection between bipartite entanglement and teleportation. Some particular cases are studied in Section V, where the relations among the teleportation and entanglement conditions are graphically represented. The dynamic of the average fidelity in terms of the attenuations are plotted in some figures. We compare symmetric and asymmetric cases, considering the effect of the gain adjustment. The results are discussed in Section VI.

II. NON-UNITY GAIN TELEPORTATION BY LOSSY CHANNELS

To study the teleportation, we should consider a system composed of three subsystems: the input signal and the pair of correlated modes shared between Alice and Bob. In the case of continuous variable systems, we can use the formalism of Wigner distributions, such that the Wigner function, $W(\mathbf{x})$, depends on ordinary variables, placed in vector form as

$$\mathbf{x} = (q_1, p_1, q_2, p_2, \dots, q_N, p_N)^T, \quad (1)$$

in the case of N subsystems. These variables are associated with the respective quantum operators,

$$\hat{\mathbf{x}} = (\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2, \dots, \hat{q}_N, \hat{p}_N)^T, \quad (2)$$

that obey the usual commutation relations, $[\hat{q}_i, \hat{p}_j] = 2i\delta_{ij}$ and $[\hat{q}_i, \hat{q}_j] = [\hat{p}_i, \hat{p}_j] = 0$ [46]. For a physical state represented by a complete density matrix ρ , the mean value of an operator $\hat{\mathcal{O}}$ is calculated by $\langle \hat{\mathcal{O}} \rangle = \text{tr}(\hat{\mathcal{O}}\rho)$. So the mean value of the operator $\hat{\mathbf{x}}$ is

$$\boldsymbol{\mu} = \langle \hat{\mathbf{x}} \rangle. \quad (3)$$

We restrict this paper to the case of Gaussian states, so that the respective Wigner function takes the general form:

$$W(\mathbf{x}) = \frac{1}{(2\pi)^{2N} \sqrt{\det \mathbf{V}}} \exp \left[-\frac{1}{2} (\mathbf{x} - \boldsymbol{\mu})^T \mathbf{V}^{-1} (\mathbf{x} - \boldsymbol{\mu}) \right], \quad (4)$$

where \mathbf{V} is the covariance matrix of the complete system, whose entries are $V_{ij} = \langle \frac{1}{2} \{ \Delta \hat{x}_i, \Delta \hat{x}_j \} \rangle$, such that $\Delta \hat{x}_i = \hat{x}_i - \mu_i$ [48].

In the teleportation protocol proposed by Braunstein and Kimble, the signal sent by Alice belongs to a set of pure coherent states, $\{|\alpha\rangle\}_A$, in which the covariance matrix of the set elements is

$$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (5)$$

and their mean values are $\langle \hat{q}_S \rangle = 2\Re(\alpha)$ and $\langle \hat{p}_S \rangle = 2\Im(\alpha)$. Assuming that the input signal states follow a central Gaussian distribution,

$$P(\alpha) = \frac{\lambda}{\pi} \exp(-\lambda|\alpha|^2), \quad (6)$$

so that we obtain an uniform distribution of coherent states taking $\lambda \rightarrow 0$.

Before signal transmission, Alice and Bob have to share each mode of a bipartite system, which is originally generated as a Gaussian state, whose covariance matrix is formed by 2x2 matrix blocks in the following way

$$V = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \quad (7)$$

where

$$A = \begin{pmatrix} Q_A & K_A \\ K_A & P_A \end{pmatrix} \quad (8)$$

is the covariance matrix of the mode designed to Alice,

$$B = \begin{pmatrix} Q_B & K_B \\ K_B & P_B \end{pmatrix} \quad (9)$$

is the covariance matrix of the mode designed to Bob, and

$$C = \begin{pmatrix} K_Q & K_1 \\ K_2 & K_P \end{pmatrix} \quad (10)$$

is the correlation matrix between such subsystems. For simplicity without losing generality, we consider that the shared bipartite system has vanishing mean-value canonical operators, because, otherwise, to restore the teleported signal, Bob only needs to modulate trivial signal displacements in phase space for compensation of the bipartite system contribution [2]. On the other hand the bipartite system modes arrive at Alice and Bob after passing through attenuation channels, which alter them according to [47]

$$V_t = \mathcal{L}(V) = L(V - I)L + I \quad (11)$$

such that V_t is the covariance matrix of the attenuated bipartite system and

$$L = \text{diag}(t_A, t_A, t_B, t_B), \quad (12)$$

where t_A and t_B are the channel transmissibilities of Alice and Bob, respectively. The submatrices of V_t are transformed by $A_t = L_A(A - I^{(2)})L_A + I^{(2)}$, $B_t = L_B(B - I^{(2)})L_B + I^{(2)}$, and $C_t = L_A C L_B$, with $L_i = \text{diag}(t_i, t_i)$, $i = \{A, B\}$ and $I^{(2)}$ is the 2x2 identity matrix.

Since each communication station is in possession of its subsystem bipartite, Alice combines the input signal with her bipartite subsystem by a beam-splitter operation and she

measures the quadratures $\hat{q}_- = (\hat{q}_A - \hat{q}_{\text{in}})/\sqrt{2}$ and $\hat{p}_+ = (\hat{p}_A + \hat{p}_{\text{in}})/\sqrt{2}$ by homodyne detection. So Alice sends the measurement outcomes, m_q e m_p , through classical channels to Bob. In his turn, Bob performs phase and amplitude modulations in his bipartite subsystem according to the received classical information, that is, $\hat{q}_{\text{out}} = \hat{q}_B - \sqrt{2}gm_q$ and $\hat{p}_{\text{out}} = \hat{p}_B + \sqrt{2}gm_p$, in which g is the gain introduced in the classical communication or modulation. We can notice that $g > 1$ makes a teleportation with amplification, whereas $g < 1$ gives a teleportation with deamplification. Cases in which the gain is different of unity are sometimes called teleamplifications [43, 44]. To $g = 1$ we get the usual protocol proposed by Braunstein and Kimble.

Taking into account the (de)amplification gain, the wanted states by Bob must be $\{|\beta\rangle = |g\alpha\rangle\}_B$. However, in a realistic situation, the actually transmitted states are described by the set of density matrices $\{\rho_{\text{out}}\}$. So we have to calculate the fidelity to the ideal task $|\alpha\rangle \rightarrow |\beta\rangle$ [34], that is,

$$F = \left[\text{tr} \sqrt{\sqrt{\rho_\beta} \rho_{\text{out}} \sqrt{\rho_\beta}} \right]^2, \quad (13)$$

where $\rho_\beta = |\beta\rangle\langle\beta|$. We can rewrite the fidelity in terms of the respective Wigner functions,

$$F = 2\pi \int W_\beta(p, q) W_{\text{out}}(p, q) dq dp, \quad (14)$$

in which $W_\beta(p, q)$ and $W_{\text{out}}(p, q)$ correspond to $|\beta\rangle$ and ρ_{out} , respectively. Proceeding with the calculation of fidelity (14) as previous references [18, 26], we obtain, then,

$$F = \frac{2 \exp \left[-\frac{1}{2} (\mathbf{x}_\beta - g\mathbf{x}_\alpha)^T E_{t,g}^{-1} (\mathbf{x}_\beta - g\mathbf{x}_\alpha) \right]}{\sqrt{\det(E_{t,g})}}, \quad (15)$$

with $\mathbf{x}_i = (2\Re(j), 2\Im(j))^T$, $j = \{\alpha, \beta\}$ and the matrix in denominator is

$$E_{t,g} = (1 + g^2)D + g^2 Z A_t Z^T - g(ZC_t + C_t^T Z^T) + B_t, \quad (16)$$

where $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. For construction of the protocol, $\beta = g\alpha$, in the way that it is trivial to calculate the mean fidelity, considering the prior distribution (6) of the input states. Therefore

$$\bar{F}(V_t; g) = \frac{2}{\sqrt{\det(E_{t,g})}}, \quad (17)$$

where the fidelity is explicitly described as function of the quantum resources, represented by V_t , and the classical communication, represented by g . This is a relevant result because

all realistic contributions, such as non-unity gain and losses of the entangled system, are included in the term $\det(E_{t,g})$. So the complete description of the teleportation protocol can be found thoroughly analyzing it.

III. ROBUST TELEPORTATION CONDITIONS

The task to perform a non-unity gain teleportation, ideally represented as $\{|\alpha\rangle\}_A \rightarrow \{|g\alpha\rangle\}_B$, has a maximal fidelity for exclusively classical resources, using measure-and-prepare strategies. As obtained in previous articles [22, 24], this classical fidelity threshold (CFT), using deterministic or probabilistic processes, is

$$F_{\text{CFT}}(g) = \frac{1}{1 + g^2}, \quad (18)$$

that is, for a genuinely quantum teleportation, the average fidelity must be strictly larger than F_{CFT} .

Hence the genuinely quantum BK teleportation condition with attenuations and non-unity gain is obtained comparing expressions (17) and (18), so that the classical regime is restricted to $\bar{F}(V_{t,g}) \leq F_{\text{CFT}}(g)$, otherwise the process is exclusively quantum. Such condition can be better analyzed if we take only the determinant in (17), so that the necessary and sufficient condition of classical regime is restricted to $\det(E_{t,g}) \geq 4(1 + g^2)^2$. This inequality depends on all entries of the bipartite system covariance matrix, thus the obtained expressions are cumbersome to derive new relations among the teleportation parameters, like g and t_i . On the other hand, we can search for symmetry properties of the average fidelity $\bar{F}(V_{t,g})$, so that a quantity preserved by some transformation could be found. In Appendix A, the following result is proved: The average fidelity $\bar{F}(V_{t,g})$ is invariant under local phase rotations of the shared bipartite system, $S_{\text{inv}} := R_{\theta_A} \oplus R_{\theta_B} \in SO(2, \mathbb{R}) \oplus SO(2, \mathbb{R})$, such that the rotation angles are constrained by $\theta_B = -\theta_A$, in which indexes A and B label the modes shared by Alice and Bob, respectively.

With this invariance property of the fidelity, we can do a phase displacement or a quadrature basis choice, to handle $\det(E_{t,g})$ without changing the average fidelity, and, therefore, maintaining the relation of the teleportation process with the CFT. Hence the classical-quantum border can be calculated as presented in Appendix B, so that we obtain the following result:

Result 1: Given a BK teleportation with amplification gain $g \geq 0$, such that the shared bipartite system suffers local attenuations t_A and t_B , and choosing a quadrature basis according to S_{inv} , then the necessary and sufficient condition to exist a classical measure-and-prepare strategy performing the same task, i.e., to the fidelity be below the CFT, is

$$\begin{aligned} W_{\text{all}} := & 2(1 + g^2) \left[(gt_A)^2(\text{tr}(A) - 2) + t_B^2(\text{tr}(B) - 2) - 2gt_At_B(K_Q - K_P) \right] + \\ & + \left[(gt_A)^2(Q_A - 1) + t_B^2(Q_B - 1) - 2gt_At_B K_Q \right] \times \\ & \times \left[(gt_A)^2(P_A - 1) + t_B^2(P_B - 1) + 2gt_At_B K_P \right] \geq 0. \end{aligned} \quad (19)$$

Of course, this proposition has a converse, namely, the sufficient and necessary condition to a genuinely quantum BK teleportation is $W_{\text{all}} < 0$. Any way, this expression has many terms, but we can rewrite it in more familiar ways. Consider the EPR-like operators

$$\hat{u} = gt_A \hat{q}_A - t_B \hat{q}_B \quad (20)$$

and

$$\hat{v} = gt_A \hat{p}_A + t_B \hat{p}_B. \quad (21)$$

So condition (19) can be written as

$$\begin{aligned} & \left[(2 - t_B^2) + (2 - t_A^2) g^2 \right] \times \\ & \times \left\{ \langle (\Delta \hat{u})^2 \rangle + \langle (\Delta \hat{v})^2 \rangle - 2 \left[(gt_A)^2 + t_B^2 \right] \right\} + \\ & + \langle (\Delta \hat{u})^2 \rangle \langle (\Delta \hat{v})^2 \rangle - \left[(gt_A)^2 + t_B^2 \right]^2 \geq 0, \end{aligned} \quad (22)$$

where the variances are calculated to EPR-like operators (20) and (21).

As the variances are non-negative quantities, we can prove that the third line in (22) is negative, if the second line in (22) is also negative. Conversely, the second line is positive, if the third line is positive as well (see Barbosa et al. [33] for a similar deduction). In addition, first line in (22) is always positive. Therefore we can split teleportation condition (22) in two weaker conditions:

Result 2: Given a BK teleportation with amplification gain $g \geq 0$, such that the shared bipartite system suffers local attenuations t_A and t_B , and choosing a quadrature basis according to S_{inv} , then a sufficient condition for its fidelity is below the CFT is

$$W_{\text{prod}} := \langle (\Delta \hat{u})^2 \rangle \langle (\Delta \hat{v})^2 \rangle - \left[(gt_A)^2 + t_B^2 \right]^2 \geq 0. \quad (23)$$

Result 3: Given a BK teleportation with amplification gain $g \geq 0$, such that the shared bipartite system suffers local attenuations t_A and t_B , then a sufficient condition in order to surpass the CFT, i.e., to be a genuinely quantum teleportation, is

$$W_{\text{sum}} := \langle (\Delta \hat{u})^2 \rangle + \langle (\Delta \hat{v})^2 \rangle - 2 \left[(gt_A)^2 + t_B^2 \right] < 0. \quad (24)$$

Again, conditions (23) and (24) have converses. In particular, from expression (23), a necessary condition for a genuinely quantum teleportation is $W_{\text{prod}} < 0$. It is very important to notice that Result 3 does not make reference to basis choice. The reason is because W_{sum} is manifestly invariant under the transformations S_{inv} , in same way that the average fidelity $\bar{F}(V_{t,g})$ is. Therefore condition (24) is as general as fidelity and CFT to characterize the teleportation process.

Conditions (23) and (24) are more convenient than condition (22) to characterize a teleportation apparatus, since it is necessary less knowledge about the shared bipartite system, that is, less covariance matrix entries for measuring. On the other hand, we have to know the gain g and the attenuation transmissibilities t_A and t_B . Nevertheless realistic scenarios, which long-distance transmissions of the correlated bipartite system would have unknown transmissibilities, are reasonable and very probable in near future. Thus let us consider an experimentalist could tune the gain of the teleportation amplification, so that the process be maintained at a level above the CFT, independent of any partial attenuation. Here it is worth to remark that the relevant attenuations are only the partial ones, $t_A; t_B > 0$, because in case of total attenuations, $t_A = 0$ or $t_B = 0$, the bipartite system shared by Alice and Bob becomes separable, in fact, the partners share no correlated system to perform teleportation, and there is no quantum process. On the contrary, in the case of partial attenuations, we have the following situations. First, one can call robust quantum teleportation a process with tunable gain and which its shared bipartite system is able to perform genuinely quantum teleportation for any partial local attenuation. Second, if there are partial attenuations that make the teleportation fidelity decrease below the CFT, the process is called fragile teleportation. Third situation is that, for any attenuation, the share bipartite system is unable to perform genuinely quantum teleportation. Such situation always occurs to separable states and to some possible entangled states.

So a relevant question for teleportation with unknown attenuations is what Gaussian bipartite systems are able to perform robust quantum teleportation. In order to find a

condition that maximally delimitates the bipartite system set able to robust quantum teleportation, we have to minimize the expression W_{sum} to satisfy the sufficient condition of genuinely quantum teleportation. It is clear that W_{sum} as function of g has a global minimum at $g_{\text{min}} = \frac{t_B(K_Q - K_P)}{t_A(\text{tr}(A) - 2)}$. Therefore, substituting g_{min} in expression (24), we obtain:

Result 4: Given a BK teleportation, there is an amplification gain, so that the sufficient condition to surpass the CFT, for any local attenuations on the shared bipartite system, is

$$W_{\text{rob}} := (\text{tr}(A) - 2)(\text{tr}(B) - 2) - (K_Q - K_P)^2 < 0. \quad (25)$$

Condition (25) is also manifestly invariant under the transformations S_{inv} (see Appendix B), so the property of a bipartite system to be able to robust teleportation follows the same generality of the characterization of a teleportation with a determinate average fidelity.

After these results, we may do some considerations. Firstly, the dependence of the average fidelity of the relative phases of the input modes has already been noted by Zhang et al. [16], in which an explicit calculation has shown that the fidelity depends on the relative phases among the shared bipartite modes and the signal teleported. So phase fluctuations insert extra noise on the output signal, degrading the average fidelity. Differently, in present article we derive the transformations that retain the invariance of fidelity, which are the phase changes with the constraint $\theta_B = -\theta_A$. Another noteworthy aspect is the symmetry properties of Gaussian bipartite systems obey a hierarchy. In terms of the phase space, the Gaussian systems are preserved by general symplectic operations $Sp(4, \mathbb{R})$ [48]; then the entanglement or separability feature is preserved by the local symplectic operations $Sp(2, \mathbb{R}) \oplus Sp(2, \mathbb{R})$ [49, 50]; in addition robust bipartite entanglement is maintained with local phase rotations $SO(2, \mathbb{R}) \oplus SO(2, \mathbb{R})$, as shown by Barbosa et al. [33]; and finally BK teleportation fidelity is invariant under local rotation with angles $\theta_B = -\theta_A$. In addition, we have also shown that the property of bipartite system able to robust quantum teleportation are invariant to this last group, the local bipartite rotations of anti-symmetric angles. Each presented operation set forms a transformation group. These groups are related by a subgroup chain, being the first the biggest group, following until the smallest group. These sequence is associate to bipartite system properties, following successively from the weakest to the strongest property, respectively.

IV. COMPARISONS WITH OTHER CRITERIA

Given the parameters g , t_A and t_B of the process, we notice that the genuinely quantum teleportation condition from (24), $W_{\text{prod}} < 0$, is formally equivalent to a bipartite entanglement condition obtained by Giovannetti et al. [51] (see also an early version by Tan [52]). However, in present article, this condition is necessary for genuinely quantum teleportation, while in the previous article [51] the condition is sufficient for entanglement. Considering the sufficient condition of genuinely quantum teleportation (24), $W_{\text{sum}} < 0$, we also notice that it is formally equivalent to sufficient bipartite entanglement conditions obtained by Simon [53] and Giovannetti et al. [51]. Considering these conditions and comparing the Simon PPT condition [50] with the complete condition of genuinely quantum teleportation (22), one can verify that Gaussian bipartite entanglement and ability to perform genuinely quantum teleportation are different properties. But, in this comparison, we do not consider any operation of optimization or adjustment of the protocol.

On the other hand, it is interesting to rewrite W_{sum} in a more customary form. Discarding trivial cases, such as the case $g = 0$, that is, when there is no classical communication, and the cases with $t_A = 0$ or $t_B = 0$, when the entangled beams are completely attenuated, resulting in clearly classical cases, we define the parameter

$$\eta = \sqrt{\frac{gt_A}{t_B}}. \quad (26)$$

So the EPR-like operators from condition (24) can be redefined as

$$\tilde{u} = \eta \hat{q}_A - \frac{1}{\eta} \hat{q}_B \quad (27)$$

and

$$\tilde{v} = \eta \hat{p}_A + \frac{1}{\eta} \hat{p}_B. \quad (28)$$

Hence condition (24) becomes formally equivalent to the sufficient entanglement condition of Duan et al. [49], namely,

$$\langle (\Delta \tilde{u})^2 \rangle + \langle (\Delta \tilde{v})^2 \rangle < 2 \left(\eta^2 + \frac{1}{\eta^2} \right). \quad (29)$$

This condition is a usual criterion to test the entanglement of continuous-variable bipartite systems. Its popularity is devoted to few necessary terms to measure, only two variances.

Thus we can consider that expression (29) is also a test valid to verify if a bipartite system is useful to teleportation task.

Result 4 presented in this paper is also very connected to an entanglement condition. In fact, condition (25) is very close to a previous result due to Barbosa et al. [33]:

Result from [33]: Given an initially entangled bipartite system, its entanglement is robust to any partial local attenuations if only if

$$W_{\text{full}} := (\text{tr}(A) - 2)(\text{tr}(B) - 2) - \text{tr}(C^T C) + 2 \det(C) \leq 0, \quad (30)$$

for Gaussian states. Condition (30) is only sufficient for non-Gaussian states.

With a suitable choice of quadrature basis, according to S_{inv} , inequality (30) reduces to condition $W_{\text{rob}} \leq 0$. Hence almost all robust Gaussian bipartite states are also Gaussian bipartite states able to robust quantum teleportation. With exception of a very small set of borderline states, namely, entangled states with $W_{\text{full}} = 0$, we have $W_{\text{full}} < 0$ and $W_{\text{rob}} < 0$ delimitate the same state set. Therefore we claim that the robust Gaussian bipartite entanglement is essentially equivalent to Gaussian bipartite system property of being able to robust quantum teleportation. We notice that the induction of the robustness property from entanglement to teleportation is also observed in article of Adesso et al. [27], where the equivalence between Gaussian bipartite entanglement and genuinely quantum teleportation, whose fidelity is optimized. Such parallelism stresses the relevance of the robustness in entangled systems.

V. SOME EXAMPLES

To clarify the relations among the teleportation conditions found in this article and the previous entanglement conditions, now we consider some specific situations in what follows. A way to verify these relationships is plotting the regions of physically possible states of the shared bipartite system as function of relevant parameters. Considering that the shared bipartite system used in teleportation is described by following symmetric covariance matrix,

$$V = \begin{pmatrix} Q & 0 & K_Q & 0 \\ 0 & P & 0 & K_P \\ K_Q & 0 & Q & 0 \\ 0 & K_P & 0 & P \end{pmatrix}, \quad (31)$$

we can plot Figure 1 as a function of $\overline{K_Q} := K_Q/Q$ and $\overline{K_P} := K_P/P$. The colorful and numbered regions indicate states with different correlation properties. In both Figures 1a and 1b, share bipartite system states able to teleportation and robust to any partial local attenuation ($0 < t_i \leq 1, i = A; B$) lie in region I (red). States able to quantum teleportation, but fragile to some partial local attenuation, are presented in region II (light blue in Figure (a) and light purple in Figure (b)). Separable states are comprised within region IV (yellow). Entangled states which are not able to genuine quantum teleportation (with fidelity below the CFT) lie in regions III (blue and light green in Figure (b)) and V in Figure (b) (purple and orange). The white regions in both Figures 1a and 1b are unphysical states, prohibited by the Robertson–Schrödinger uncertainty relation and the purity [54, 55], $0 < (\det(V))^{-2} \leq 1$. The plots are limited to $|\overline{K_Q}|, |\overline{K_P}| \leq 1$, due to the Schwartz inequality. Figure I presents two situations for comparison of a symmetrized/optimized teleportation (Figure 1a) and a non-optimized teleportation (Figure 1b). In the Figure 1a, the attenuations and gain are such that $\eta = gt_A/t_B = 1$. So in this case, the border between regions I and II is given by coincident conditions (24) and (25), (the border on $W_{\text{sum}} = 0$ and $W_{\text{rob}} = W_{\text{full}} = 0$); regions II and III are delimited by condition (22) (border on $W_{\text{all}} = 0$), and the border between regions III and IV is determined by condition (23) (border on $W_{\text{prod}} = 0$) and the Simon PPT condition, which are coincident in this particular case. However, the situation is more complex in Figure 1b, where it is taken $\eta = gt_A/t_B = 0.65$ and $t_B = 1$. Here condition to $W_{\text{sum}} = 0$ is displaced to delimit regions I and II, and condition $W_{\text{rob}} = W_{\text{full}} = 0$ is maintained, delimiting regions III and V, because it does not depend on the parameters g , t_A and t_B . Condition $W_{\text{all}} = 0$ is also displaced to delimit regions II and V. At last, condition (23) and the Simon PPT condition are no longer coincident. PPT condition stays set apart region IV from the other states, but condition $W_{\text{prod}} = 0$ is displaced to separate regions (III/V) a from b. We can notice that regions useful to teleportation are reduced if g is not optimized. However, condition (25) establishes what shared bipartite states can become able to robust teleportation, given a suitable gain, extending red region I in Figure 1b to regions II and V. Therefore the robust teleportation state regions are the same in Figures 1a and 1b.

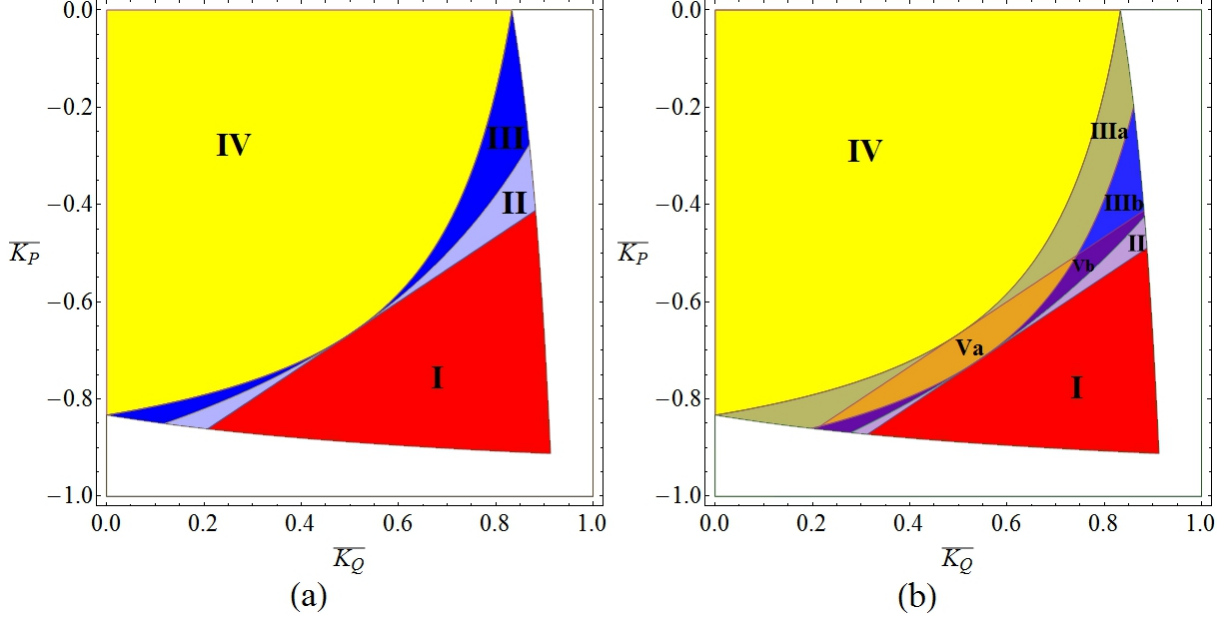


Figure 1: The space of states of the Gaussian bipartite system with covariance matrix (31) is presented as a function of $\overline{K_Q} := K_Q/Q$ and $\overline{K_P} := K_P/P$. In Figure (a), the teleportation is performed tuning gain as $gt_A/t_B = 1$. Bipartite system states able to robust quantum teleportation lie in region I (red). States able to quantum teleportation, but fragile to some partial local attenuation, are presented in region II (light blue). Entangled states which are not able to genuine quantum teleportation (with fidelity below the CFT) lie in region III (blue). Separable states are comprised within region IV (yellow). In Figure (b), the teleportation is performed with $t_B = 1$ and $gt_A/t_B = 0.65$. As in Figure (a), robust quantum teleportation states lie in region I (red), fragile quantum teleportation states lie in region II (light purple), and separable states lie in region IV (yellow). Moreover, as g is not optimized, regions IIIa/b and Va/b represent the entangled states unable to genuine quantum teleportation. However, the gain g can always be tuned, so that region I is maximized, expanding on regions II and V. In both Figures, the white region represents unphysical states.

In addition, we can study specific states of the shared bipartite system, to visualize the dynamics of the average fidelity in terms of the local attenuations and the gain. In Figure 2, it is plotted the teleportation fidelity as function of the attenuation transmissibilities t_A and t_B , for unity gain and the shared bipartite system being initially in a symmetric two-mode squeezed state, such that the covariance matrix entries are $Q_A = P_A = Q_B = P_B = \cosh(2r)$,

$K_Q = -K_P = \sinh(2r)$, and null in other cases. In Figure 2, the CFT is represented by the thick straight lines. The clearer coned region is where the teleportation surpasses the CFT. A very similar plot was obtained by He et al. [44]. In that study, however, the delimited coned region is concerning the so-called secure teleportation, that is, teleportation with the fidelity above the telecloning threshold. Hence in that paper the mentioned region is narrower than the present study, because the telecloning threshold is $2/3$, for unity gain. This dynamic of the fidelity in terms of the transmissibilities is persistent for symmetric bipartite states. But as we shall see, this is not true, when considering asymmetric states.

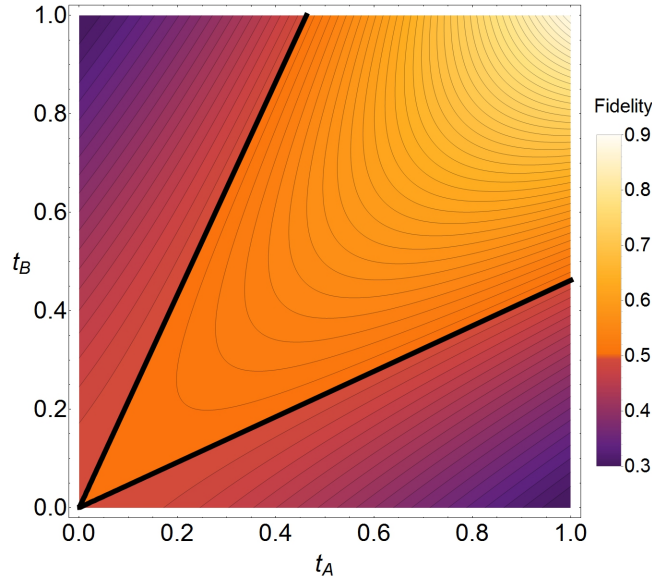


Figure 2: Fidelity as a function of the transmissibilities t_A and t_B . The quantum teleportation is possible inside the coned region. In this plot, the gain is $g = 1$ and the entangled bipartite system is a symmetric two-mode squeezed state with squeezing parameter $r = 1$.

With a direct verification of condition (25), the state considered in Figure 2 is able to robust quantum teleportation, although there are non-null values of t_A and t_B so that the fidelity is below the CFT. However, this case is fixed to $g = 1$, and the definition of robust quantum teleportation is that the teleportation process, whose amplification gain can be tuned, has an average fidelity larger than the CFT for any local partial attenuation. Hence the coned region in Figure 2 can be displaced to cover all $t_A \times t_B$ plane, as the gain is tuned to this purpose. This is observed in Figure 3, in which is used the same original state of Figure 2. In Figure 3a, the gain is $g = 0.5$, so the coned region is shifted toward the axis t_A . As $g \rightarrow 0$, the coned region continuously approaches to this axis. A side effect is that the

average fidelity increases, as well as the CFT. Opposite effects are found in the case of gain $g = 2.5$, in Figure 2b, in which the coned region is shifted toward the axis t_B . As $g \rightarrow \infty$, the coned region continuously approaches to this axis, and the average fidelity and the CFT decrease. These characteristics are found in all sufficiently symmetrical states, which for many practical purposes are interesting and useful for communication and processing of the quantum information.

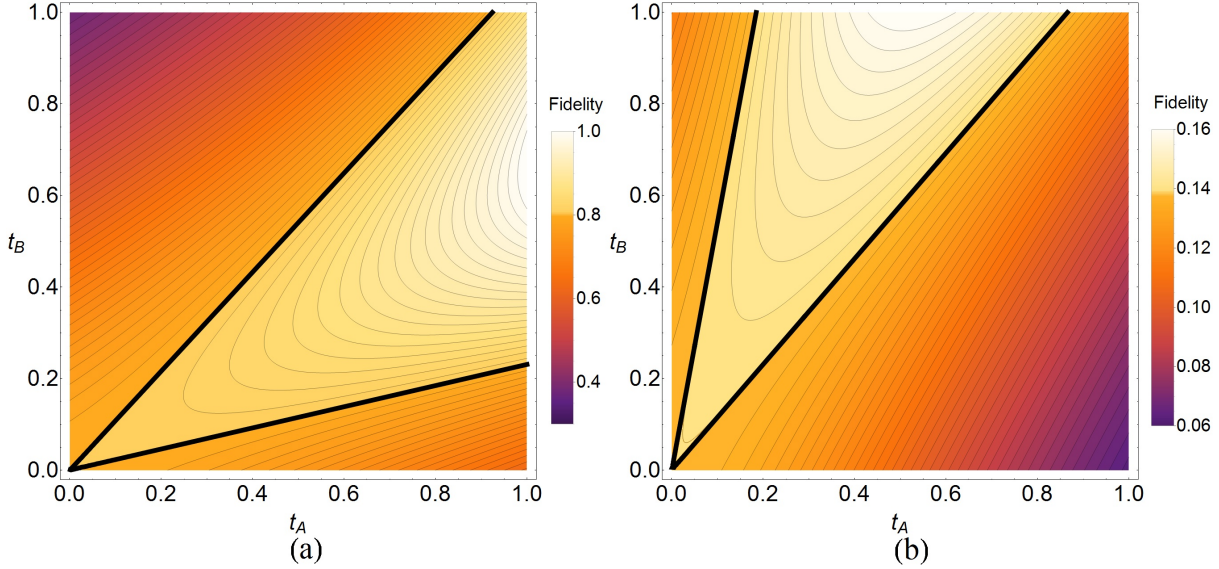


Figure 3: Fidelity as a function of the transmissibilities t_A and t_B . The quantum teleportation is possible inside the coned regions. In these plots, the entangled bipartite system is same as in Figure 2. In Figure (a), the gain is $g = 0.5$. In Figure (b), the gain is $g = 2.5$. We notice that this bipartite system is able to robust quantum teleportation, because the coned region sweeps all plane $t_A \times t_B$, except regions of total attenuation ($t_i = 0$), as the gain is tuned, inside the range $g \in (0, \infty)$.

Beyond the symmetric states, it is also important to fully characterize all possible scenarios for bipartite generic Gaussian systems, because in further applications of the teleportation, the modes sent to Alice and Bob could be ill generated and undergo unknown attenuations. Therefore we have to focus the asymmetric bipartite states as well. Consider-

ing a case such that

$$V = \begin{pmatrix} 2.1 & 0 & 1.9 & 0 \\ 0 & 2.6 & 0 & -0.7 \\ 1.9 & 0 & 2.2 & 0 \\ 0 & -0.7 & 0 & 2.4 \end{pmatrix}, \quad (32)$$

and with unity gain, one can plot Figure 4, in which the trick curve assigns the CFT. We notice that the genuinely quantum teleportation is only maintained for transmissibility values close to 1, observed in the region surrounded in the upper right corner of Figure 4. For non-unity gain values, the genuinely quantum teleportation region tends to decrease, so that the quantum region cannot cover all $t_A \times t_B$ plane, even tuning the gain. So this is a typical case of fragile bipartite states for quantum teleportation.

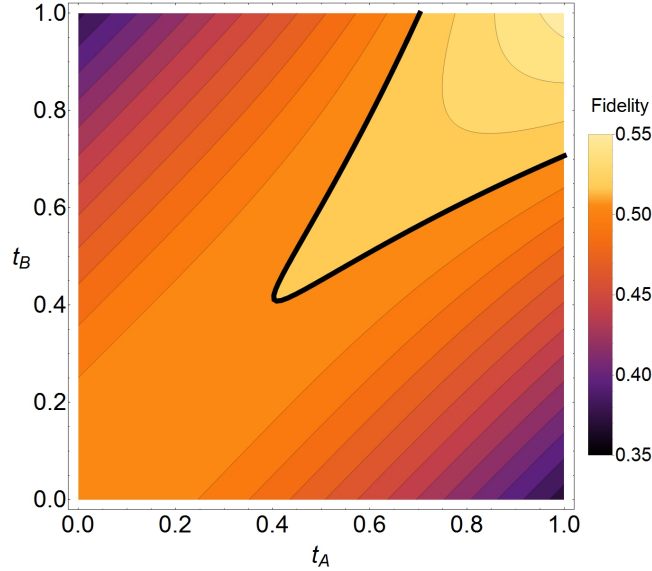


Figure 4: Fidelity as a function of the transmissibilities t_A and t_B . The quantum teleportation is possible inside the upper right region. In this plot, the gain is $g = 1$ and the entangled bipartite system is an asymmetric two-mode state with covariance matrix given by Eq. (32). This example presents a fragile bipartite system to perform teleportation, that is, even tuning the gain, the quantum teleportation region does not cover all values of $0 < t_i \leq 1$.

Finally, there are entangled bipartite states that, together the separable states, is not useful for a teleportation with fidelity above the CFT. Once these cases are trivial, a respective figure is not presented.

VI. DISCUSSION

In this article, we have studied the details of the relationship between the gain of the classical communication and the losses of the correlated bipartite system shared by Alice and Bob, in the teleportation of coherent state signals. We have found several conditions that characterize the BK teleportation as functions of the its parameters, namely, the bipartite system covariance matrix, the gain, and the attenuation transmissibilities. These conditions provide criteria to determinate if the teleportation process is successful for its fidelity being above the classical threshold (CFT) of measure-and-prepare methods. From that, given a suitable gain tuning, it was obtained a condition to the shared bipartite system to be able to execute BK teleportation robust to local partial attenuations. Such condition to robust quantum teleportation was verified to be essentially equivalent to the robust bipartite entanglement condition, showing that the robustness of the entanglement is induced in the teleportation task. Along the presented derivations, fidelity symmetry properties were found, revealing that the main results obey the same transformation invariance of the average fidelity. It is expected that these findings are helpful to future long-distance teleportation implementations, in which one wants to generate a shared bipartite system useful for this task, independently of unknown attenuations.

As an initial motivation was to study the realistic conditions to perform the teleportation, this article has a natural path to be generalized, considering other dissipative dynamics of the quantum channels of the bipartite system, for example, phase losses or atmospheric turbulence, as it was studied by Bohmann et al. [56]. Other possible extension of the presented studies treats about the teleported signal. We have studied the case of transmitted signals belonging to an uniform distribution of coherent states. However we can consider other sets of coherent states, so that the teleportation task is more challenging and diverse, including more complex dynamics combined with quantum channel noise [57]. Following in this way, future progresses can generalize the present article to may other states of the input signal, for example, general Gaussian pure states, in particular squeezed states, since the respective CFTs have already been obtained by Chiribella and Adesso [39], or qbits and single photons [58], whose implementations have been accomplished in recent years [40–42]. At last, further connections between the robust entanglement and teleportation conditions and other quantum properties, like EPR correlations, will possibly be established, as well

as the secure teleportation is connected to EPR steering [44].

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Appendix A: Symmetry Transformations of Fidelity

The average fidelity of the teleportation of coherent states with classical channel gain $g \geq 0$ and independent attenuations of the bipartite system modes is given by $\bar{F}(V_t; g) = \frac{2}{\sqrt{\det(E_{t,g})}}$ (see, e.g., [26]). Regardless of symmetry considerations, the determinant is

$$\begin{aligned} \det(E_{t,g}) = & 4(1 + g^2)^2 + \\ & + 2(1 + g^2) \left[(gt_A)^2(Q_A + P_A - 2) + t_B^2(Q_B + P_B - 2) - 2gt_At_B(K_Q - K_P) \right] + \\ & + \left[(gt_A)^2(Q_A - 1) + t_B^2(Q_B - 1) - 2gt_At_BK_Q \right] \times \\ & \times \left[(gt_A)^2(P_A - 1) + t_B^2(P_B - 1) + 2gt_At_BK_P \right] + \\ & - \left[(gt_A)^2K_A - t_B^2K_B + 2gt_At_B(K_1 - K_2) \right]^2. \end{aligned} \quad (\text{A1})$$

Managing this expression is difficult, particularly the correlations in the last line of (A1). However cumbersome derivations can be bypassed using symmetry transformations which preserve $\bar{F}(V_{t,g})$ invariant. As we are studying the shared bipartite system properties in Gaussian teleportation, the transformations must preserve the Gaussian feature of the system. So the transformations belong to unitary linear Bogoliubov maps [48]. Furthermore we must seek operations that, at least, have the basic property of preserving entanglement, which is reserved for the local maps. These transformations correspond to affine symplectic maps acting on the phase space. To covariance matrices, these maps operate as $V \rightarrow SVS^T$, where S is a $2N \times 2N$ real symplectic matrix, so that N is the number of subsystems. Some of its properties are $S\Omega S^T = \Omega$, such that $[\hat{\mathbf{x}}, \hat{\mathbf{x}}^T] = 2i\Omega$, and $\det(S) = 1$. In the case of a bipartite system shared by Alice and Bob, the general symplectic transformations are $S_{AB} \in Sp(4, \mathbb{R})$, using the index A to Alice and B to Bob. Then we must seek a symplectic

operation, S_{inv} , on the covariance matrix of the shared bipartite system, such that

$$\bar{F}(V_t; g) = \bar{F}(S_{\text{inv}} V_t S_{\text{inv}}^T; g). \quad (\text{A2})$$

As the local transformations are a subgroup of $Sp(4, \mathbb{R})$, that is, $S_{\text{local}} = S_A \oplus S_B \in Sp(2, \mathbb{R}) \oplus Sp(2, \mathbb{R})$. For each single subsystem, the local symplectic transformations $S_i \in Sp(2, \mathbb{R})$, $i = A, B$, can always be written as a product of three special maps,

$$S_i = R_\theta Y_r R_\phi, \quad (\text{A3})$$

where Y_r is a squeezing operation and R_θ and R_ϕ are space-phase rotations.

Moreover, S_{inv} must be independent of the quantum channels, through which the bipartite system is delivered to Alice and Bob, so

$$S_{\text{inv}} V_t S_{\text{inv}}^T = S_{\text{inv}} \mathcal{L}(V) S_{\text{inv}}^T = \mathcal{L}(S_{\text{inv}} V S_{\text{inv}}^T). \quad (\text{A4})$$

As it is shown in the paper by Barbosa et al. [33], in the case of Gaussian attenuations, equation (A4) is valid only if $S_i S_i^T = I$, to each mode i . This condition limits S_{inv} to the phase-space local rotation group, $S_{\text{inv}} = R_A \oplus R_B \in SO(2, \mathbb{R}) \oplus SO(2, \mathbb{R})$. For each mode, the rotations have the property of $RZ = ZR^T$, so we obtain

$$\begin{aligned} E_{t,g} &= (1 + g^2)D + g^2 Z R_A A_t R_A^T Z^T - g(Z R_A C_t R_B^T + R_B C_t^T R_A^T Z^T) + R_B B_t R_B^T = \\ &= (1 + g^2)D + R_A^T (g^2 Z A_t Z^T) R_A - R_A^T g Z C_t R_B^T - R_B g C_t^T Z^T R_A + R_B B_t R_B^T = \\ &= R_\theta [(1 + g^2)D + g^2 Z A_t Z^T - g(Z C_t + C_t^T Z^T) + B_t] R_\theta^T, \end{aligned} \quad (\text{A5})$$

where $R_B = R_A^T = R_A^{-1} = R_\theta$ and $\det(R_\theta) = 1$. Therefore the fidelity is invariant under local phase rotations of the shared bipartite system, constrained to $\theta_B = -\theta_A = \theta$, namely,

$$S_{\text{inv}} \in \{R_A(\theta_A) \oplus R_B(\theta_B) \in SO(2, \mathbb{R}) \oplus SO(2, \mathbb{R}) | \theta_A = -\theta_B\}. \quad (\text{A6})$$

Appendix B: Properties of $\det(E_{t,g})$ under S_{inv}

Since we have established the mapping on V_t that keeps invariant $\bar{F}(V_t; g)$, we can show equation (A1) has terms which are not relevant to $\bar{F}(V_t; g)$, given a suitable choice of the shared system quadrature basis. We will show that there is always an S_{inv} , such that $V \mapsto V'$ and

$$\left[(gt_A)^2 K_A - t_B^2 K_B + 2gt_A t_B (K_1 - K_2) \right] \mapsto \left[(gt_A)^2 K'_A - t_B^2 K'_B + 2gt_A t_B (K'_1 - K'_2) \right] = 0. \quad (\text{B1})$$

To obtain this, we explicitly calculate the prime entries in expression (B1), under the transformation $S_{\text{inv}} = \begin{pmatrix} \cos \theta_A & \sin \theta_A \\ -\sin \theta_A & \cos \theta_A \end{pmatrix} \oplus \begin{pmatrix} \cos \theta_B & \sin \theta_B \\ -\sin \theta_B & \cos \theta_B \end{pmatrix}$. So

$$\begin{aligned} & (gt_A)^2 \left[\frac{1}{2}(Q_A - P_A) \sin(2\theta_A) + K_A \cos(2\theta_A) \right] + \\ & -t_B^2 \left[\frac{1}{2}(Q_B - P_B) \sin(2\theta_B) + K_B \cos(2\theta_B) \right] + \\ & + 2gt_A t_B [(K_Q + K_P)(\cos \theta_A \sin \theta_B - \sin \theta_A \cos \theta_B) + \\ & + (K_1 - K_2)(\cos \theta_A \cos \theta_B + \sin \theta_A \sin \theta_B)] = 0. \end{aligned} \quad (\text{B2})$$

To S_{inv} , we must have $\theta_A = -\theta_B = \theta$. Collecting θ , we get

$$\theta = \frac{1}{2} \arctan \left[\frac{(gt_A)^2 K_A - t_B^2 K_B + 2gt_A t_B (K_1 - K_2)}{\frac{1}{2}(gt_A)^2 (Q_A - P_A) + \frac{1}{2}t_B^2 (Q_B - P_B) - 2gt_A t_B (K_Q + K_P)} \right]. \quad (\text{B3})$$

As the arctan domain is all \Re and arctan is a many-valued function, so there are always multiple values to θ . Therefore, we can always write the covariance matrix in a phase-space basis, so that $[(gt_A)^2 K_A - t_B^2 K_B + 2gt_A t_B (K_1 - K_2)] = 0$, keeping $\bar{F}(V_t; g)$ invariant. Using such phase-space basis, the condition (19) can be established.

Another important property of V under transformation S_{inv} is that conditions (24) and (25) are also manifestly invariant. To show this, we must notice that $\text{tr}(A)$, $\text{tr}(B)$ and $(K_Q - K_P)$ are invariant for any R_A and R_B . As we can write $W_{\text{sum}} = (gt_A)^2 (\text{tr}(A) - 2) + t_B^2 (\text{tr}(B) - 2) - 2gt_A t_B (K_Q - K_P)$, its invariance is clear. The same is valid to W_{rob} .

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